Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

4859577875

ADDITIONAL MATHEMATICS **4037/14**

Paper 1 **May/June 2021**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

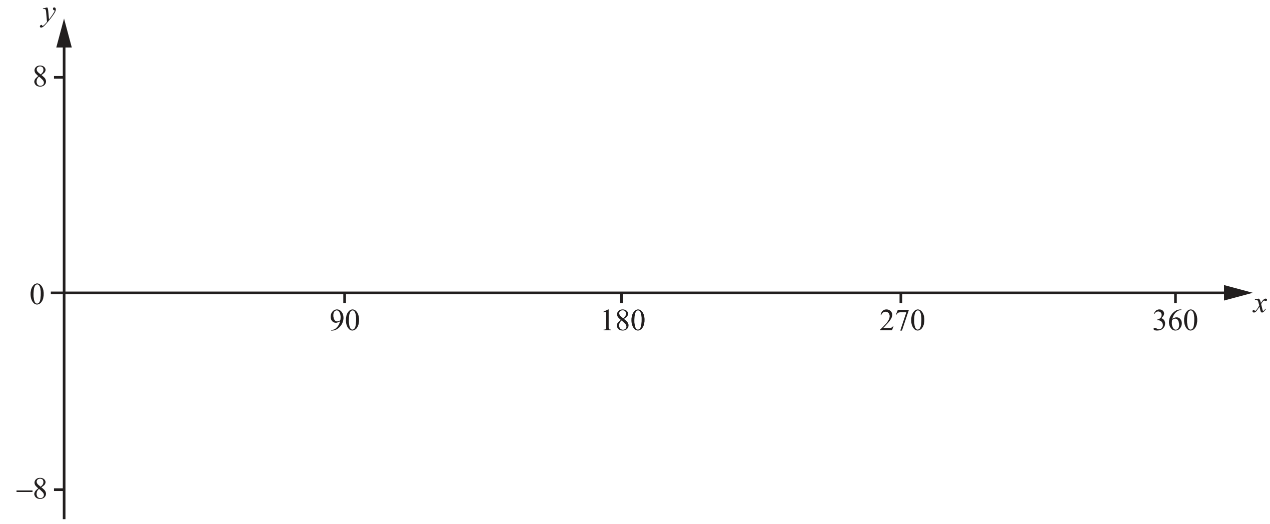
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

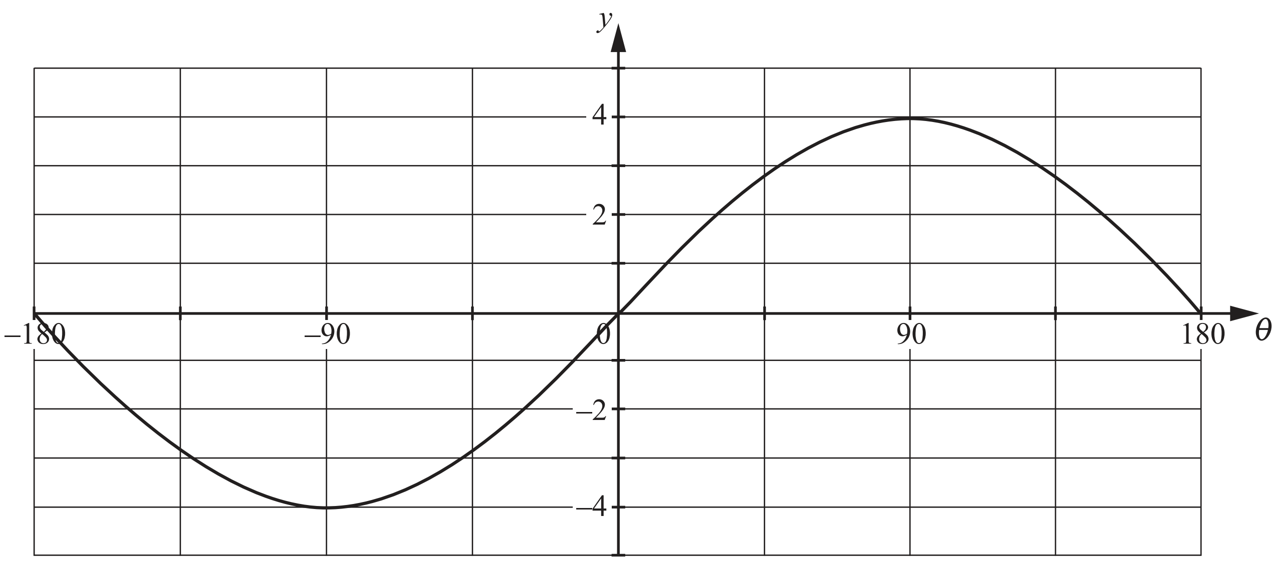
$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** On the axes below, sketch the graph of $y = 6\cos 2x - 1$ for $0° \leqslant x \leqslant 360°$.



[3]

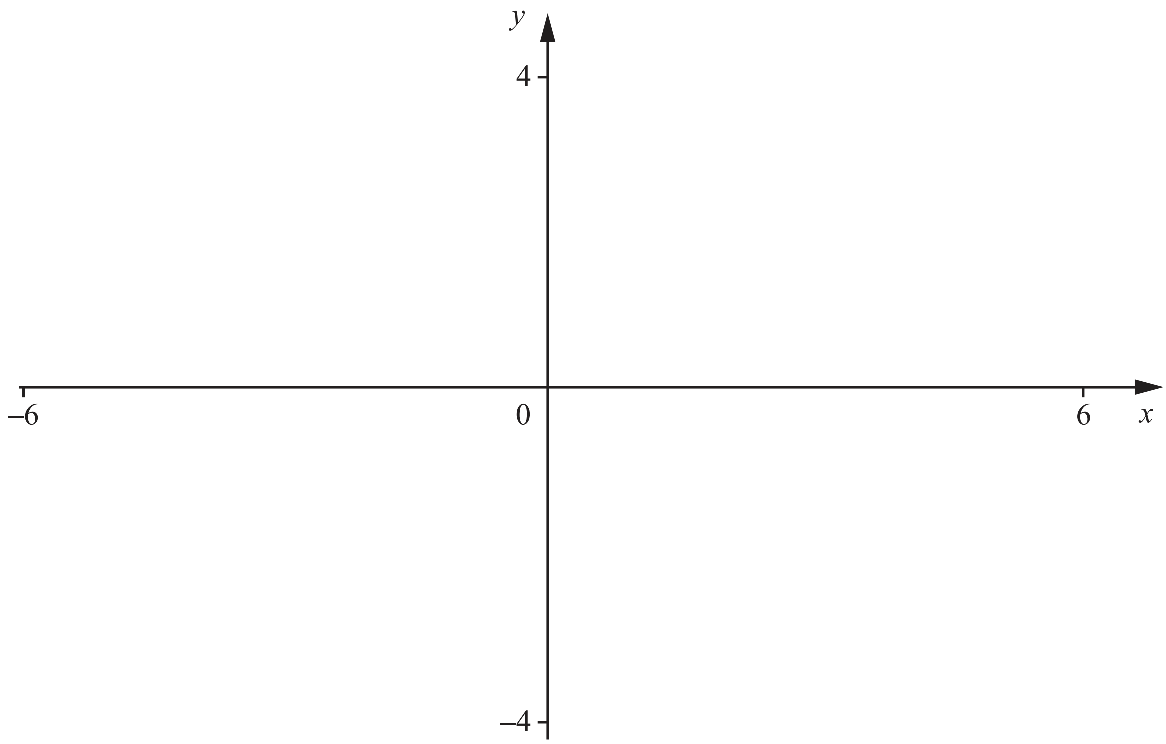
(b) The graph of $y = a + b\sin c\theta$ for $-180° \leqslant \theta \leqslant 180°$ is shown below.



Write down the value of each of the constants a, b and c. [2]

a = ... b = ... c = ...

2 **(a)** On the axes below, sketch the graphs of $y = |x - 3|$ and $y = \left|\frac{2}{5}x\right|$, giving the coordinates of the points where the graphs meet the axes. [3]



(b) Solve the equation $\left|\frac{2}{5}x\right| = |x - 3|$. [2]

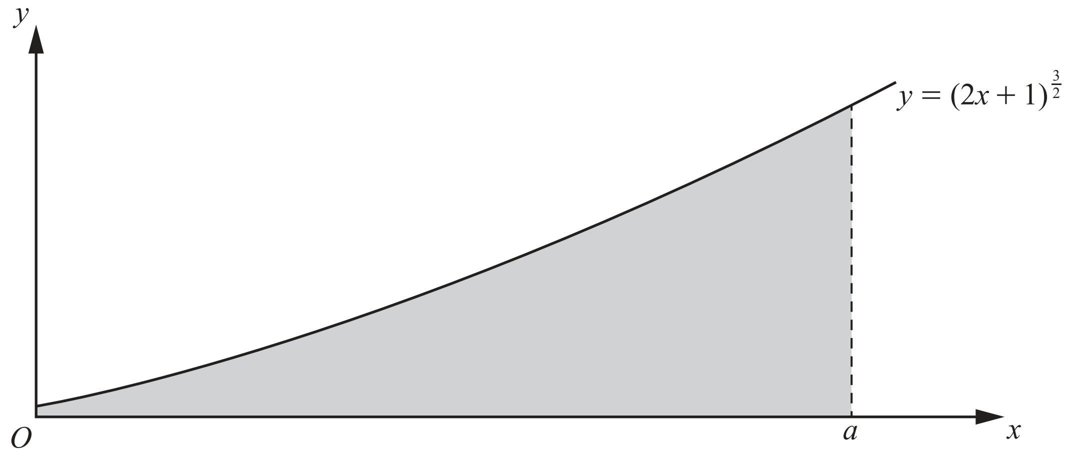
3 **(a)** Find the first 3 terms in the expansion, in ascending powers of x, of $(a-3x)^{10}$, where a is a constant. [3]

(b) Given that a is positive and that the three terms found in **part (a)** can also be written as $p+qx+\frac{405}{256}x^2$, find the value of each of the constants a, p and q. [3]

4 **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}(2x+1)^{\frac{5}{2}}$. [2]

(b) Hence find $\int (2x+1)^{\frac{3}{2}}\,\mathrm{d}x$. [2]

(c)



The diagram shows the graph of the curve $y = (2x+1)^{\frac{3}{2}}$ for $x \geqslant 0$. The shaded region enclosed by the curve, the axes and the line $x = a$ is equal to 48.4 square units. Find the value of a, showing all your working. [3]

5 **(a)** A 5-digit number is to be formed from the digits 2, 5, 6, 7 and 9. Each digit may only be used once.

(i) Find the number of different 5-digit numbers that can be formed. [1]

(ii) Find the percentage of these numbers that are odd. [2]

(b) 12 people are placed at random in 3 groups of 4 people each. Find the number of ways that this can be done. [3]

6 **(a)** Solve the simultaneous equations

$$\log_a(x+y) = 0,$$
$$\log_a(x+1) = 2\log_a y.$$ [4]

(b) Given that $\log_p q^2 \times \log_q p^3 = A$, find the value of the constant A. [3]

7 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 8\sin 2x$. The curve has a gradient of 6 at the point $\left(\frac{\pi}{2}, 4\pi\right)$.

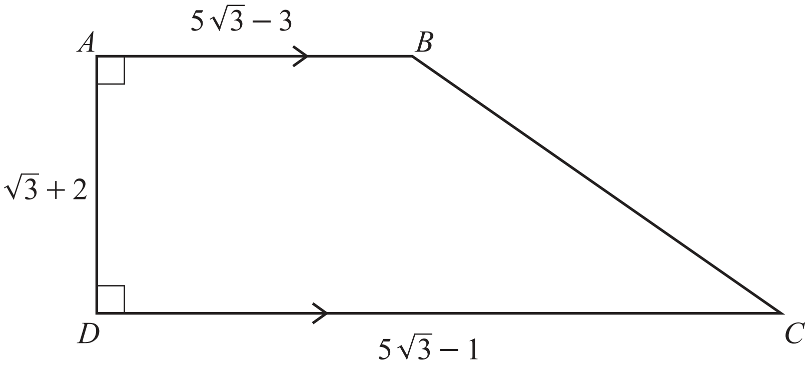
Find the equation of the curve. [8]

8 The polynomial $\mathrm{p}(x)$ is $ax^3 + bx^2 + 7x + 1$, where a and b are integers. It is given that $2x + 1$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $x - 3$ there is a remainder of 175.

(a) Find the value of a and of b. [5]

(b) Using your values of a and b from **part (a)**, find the remainder when $\mathrm{p}'(x)$ is divided by $x - 1$. [3]

9 In this question all lengths are in centimetres.
Do not use a calculator in this question.



The diagram shows the trapezium $ABCD$, where $AB = 5\sqrt{3}-3$, $DC = 5\sqrt{3}-1$ and $AD = \sqrt{3}+2$.

(a) Find the area of $ABCD$, giving your answer in the form $a+b\sqrt{3}$, where a and b are integers. [3]

(b) Given that angle $BCD = \theta$ radians, find the value of $\cot\theta$ in the form $c + d\sqrt{3}$, where c and d are integers. [3]

(c) Using your answer to **part (b)**, find the value of $\text{cosec}^2\theta$ in the form $e + f\sqrt{3}$, where e and f are integers. [2]

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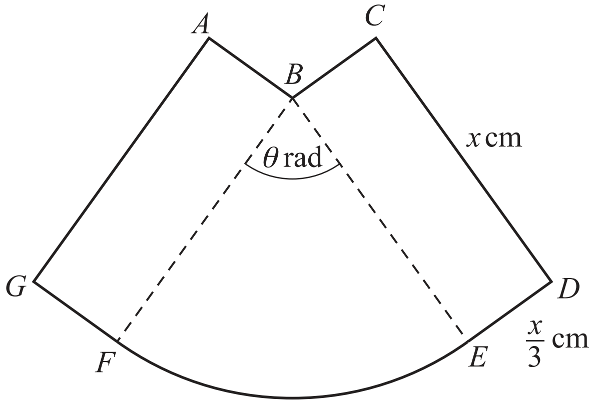

The diagram shows the figure *ABCDEFG*, where *ABFG* and *BCDE* are rectangles of length x cm and width $\frac{x}{3}$ cm. The sector *BFE* of the circle, centre *B*, radius x cm, has an angle of θ radians. It is given that the area of *BFE* is $2\,\text{cm}^2$.

(a) Show that the perimeter, P cm, of the figure *ABCDEFG* is given by $P = \frac{10x}{3} + \frac{4}{x}$. [5]

(b) Given that x can vary, find the minimum value of P in the form $q\sqrt{30}$, where q is a rational number. [4]

(c) Verify that P is a minimum. [1]

Question 11 is printed on the next page.

11 The tangent at the point where $x = 1$ on the curve $y = 6x\ln(x^2 + 1)$ intersects the y-axis at the point P. This tangent also intersects the line $x = 2$ at the point Q. A line through P, parallel to the x-axis, meets the line $x = 2$ at the point R. Find the exact area of triangle PQR. [10]